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# ECONOMIC ANALYSES FOR THE EVALUATION OF IS PROJECTS

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## ABSTRACT

Information system projects usually have numerous uncertainties and several conditions of risk that make their economic evaluation a challenging task. Each year, several information system projects are cancelled before completion as a result of budget overruns at a cost of several billions of dollars to industry. Although engineering economic analysis offers tools and techniques for evaluating risky projects, the tools are not enough to place information system projects on a safe budget/selection track. There is a need for an integrative economic analysis model that will account for the uncertainties in estimating project costs benefits and useful lives of uncertain and risky projects. The fuzzy set theory has the capability of representing vague data and allows mathematical operators and programming to be applied to the fuzzy domain. The theory is primarily concerned with quantifying the vagueness in human thoughts and perceptions. In this article, the economic evaluation of information system projects using fuzzy present value and fuzzy B/C ratio is analyzed. A numerical illustration is included to demonstrate the effectiveness of the proposed methods.

**Keywords:** *Information Systems; Project Evaluation; Fuzzy Present Value Analysis; B/C Ratio; Fuzzy Numbers* .

## 1. INTRODUCTION

The term information system (IS) sometimes refers to a system of persons, data records and activities that process the data and information in an organization, and it includes the organization's manual and automated processes. The study of information

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systems originated as a sub-discipline of computer science in an attempt to understand and rationalize the management of technology within organizations. It has matured into a major field of management that is increasingly being emphasized as an important area of research in management studies, and is taught at all major universities and business schools in the world.

Information Systems has a number of different areas of work:

- Information Systems Strategy
- Information Systems Management
- Information Systems Development

Each of which branches out into a number of sub disciplines, that overlap with other science and managerial disciplines such as computer science, pure and engineering sciences, social and behavioral sciences, and business management.

From prior studies and experiences with information systems there are at least four classes of information systems:

- Transaction processing systems
- Management information systems
- Decision support systems
- Expert systems

The need for more information system projects continues to grow as we continue to witness rapid advances in information technology. In today's increasingly competitive business climate, information system (IS) plays a major role in the success of companies. In the latest decades, significant productivity improvements have been experienced in business by IS implementations. IS implementations are widely considered as the main cause of these increases. IS implementations and advantages can be summarized as operation speed, data and data generation consistency and widely distribution and accessibility of information.

Information system projects have numerous uncertainties and several distinguished characteristics that make their analyses challenging tasks. Information system projects have several characteristics, including a high level of professionalism, high technological base, time sensitivity of projects, interdependency among various projects, and intense collaboration of different project stakeholders. They are also subject to several conditions of uncertainty as a result of the combination of some or all of these characteristics.

The decision to invest in an information system requires proven economic analysis. Economic analysis offers tools and techniques for evaluating risky projects, including information system projects. Those tools are not sufficient to place information system projects on a safe budget track. Some of the underlying problems are managerial, technical, and, of course, inappropriate economic evaluation techniques. Inappropriate economic evaluation techniques could lead to the selection of wrong projects, under budgeting or over budgeting. These indicate that there is a need for an integrated approach for evaluating information system projects.

Estimating either the benefits or the costs of an IS project is usually a difficult task because of several reasons. Some of the reasons are the uniqueness of each project, lack of historical data for cost estimation, indefinite streams of costs and benefits, presence of several intangible benefits that are not easy to quantify, the tendency to underestimate costs beyond the project life, and lack of a technique for handling delayed benefits. Other reasons are high capital cost, difficulty in predicting benefits over extended periods, and performance uncertainty of the new technology. Therefore, information system project costs and benefits estimates are neither deterministic nor stochastic; they are usually fuzzy because there are elements of vagueness in their estimations. This imprecision is as a result of intense human subjectivity involved and the lack of adequate knowledge in the execution of the projects. Hence, the conventional techniques are not enough for evaluating IS projects. The implication of using any of these techniques for information system projects as if they were like any other privately funded projects has resulted in either choosing the wrong project or underestimating project costs and benefits (Omitaomu and Badiru, 2007).

The objective of this article is to present fuzzy models for evaluating information system projects based on their present value and B/C ratio using a fuzzy modeling technique. These models have the potential of enhancing the selection process of an IS project that meets organizational objectives and maximizes its benefits to the organization.

The rest of the paper is organized as follows. Section 2 presents a literature review on fuzzy cash flow analysis and fuzzy investment evaluation. Section 3 explains fuzzy numbers. Section 4 includes fuzzy present value analysis. Section 5 presents fuzzy benefit / cost ratio analysis. Section 6 includes some defuzzification methods. Section 7 gives a numerical example, which is applied in both fuzzy PV analysis and fuzzy B/C analysis. Section 8 finally concludes the results and suggestions for further research.

## 2. LITERATURE REVIEW

The works related to the fuzzy cash flows and fuzzy investment evaluations in the literature are as follows (Kahraman, 2008):

Buckley (1987: 257) developed fuzzy analogues of the elementary compound interest problems in the mathematics of finance and used fuzzy present value and fuzzy future value of fuzzy cash amounts and also fuzzy interest rates, over *n* periods where *n* may be crisp or fuzzy. In 1992, Buckley (1992: 289) applied the new solution procedure fuzzy equations in economics and finance: Leontief's input-output model; Internal rate of return; Dynamic supply-demand model.

Calzi (1990: 265) worked on the fuzzy mathematics of finance and provided conditions for a consistent fuzzy extension of present and future value. In his study, Gupta (1993: 175) showed that under certain conditions fuzzy information about cash flows can be approximated by normal probability distribution. As an alternative to conventional cash flow models, Chiu and Park (1994: 113) proposed an engineering economic decision model in which uncertain cash flows and discount rates are specified as triangular fuzzy numbers. They worked also on the capital budgeting problems under risk where all the information is known with probability distributions (Chiu and Park, 1998: 125). Another study on risk evaluation system for capital investment was conducted by Liang and Song (1994: 391). Their risk evaluation system was computer-aided.

Karsak (1998: 331) presented formulations for the fuzzy payback method and the fuzzy duration analysis, specifying cash flows and discount rate as triangular fuzzy numbers. Terceno et al. (2003:263) showed how Fuzzy Set Theory can be used in investment analysis when the investor has only subjective estimates based on his experience or knowledge about the future cash flows of the investments, the discount rate, etc. In their study, Kahraman and Ulukan (1997: 1451) derived fuzzy present value and fuzzy future value for the case of continuous compounding. Kahraman et al. (2000: 45) used the fuzzy benefit-cost (*B*/*C*) ratio method to justify manufacturing technologies. After calculating the *B*/*C* ratio based on fuzzy equivalent uniform annual value, they compared two assembly manufacturing systems having different life cycles.

Dimitrovski (2000: 283) presented an approach for including non-statistical uncertainties in engineering economic analysis, particularly utility economic analysis, by modeling uncertain variables with fuzzy numbers. Kuchta (2000: 367) aimed to propose a practical tool of incorporating uncertainty into capital budgeting in its simplest form. In another study, Kuchta (2001: 164) proposed a model of selecting a subset of a collection of indivisible projects which maximizes the global Net Present Value.

Kahraman et al. (2002: 57) developed the formulas for the analyses of fuzzy present value, fuzzy equivalent uniform annual value, fuzzy future value, fuzzy benefit–cost ratio, and fuzzy payback period and gave some numerical examples. In their study in 2003, Kahraman et al. (2003:101) applied the dynamic programming to the situation where each investment in the set has the following characteristics: the amount to be invested has several possible values, and the rate of return varies with the amount invested. To obtain a sensible result in quantifying the manufacturing flexibility in computer integrated manufacturing systems, the paper of Kahraman et al. (2004: 77) proposed some fuzzy models based on fuzzy present value.

Tolga et al. (2005: 89) worked on creating an Operating System (OS) selection framework for decision makers (DMs). Since DMs have to consider both economic and non-economic aspects of technology selection, both factors have been considered in the developed framework. The economic part of the decision process has been developed by Fuzzy Replacement Analysis. The article of Liou and Chen (2006: 19) proposed a fuzzy equivalent uniform annual value (fuzzy EUAV) method to assist practitioners in evaluating investment alternatives utilizing the theory of fuzzy sets. Triangular fuzzy numbers (TFNs) are used throughout the analysis to represent uncertain cash flows and discount rates.

In his paper, Huang (2007: 149) studied capital budgeting problem with fuzzy investment outlays and fuzzy annual net cash flows based on credibility measure. Net present value (NPV) method is employed, and two fuzzy chance-constrained programming models for capital budgeting problem are provided. The paper of

Carmichaela and Balatbat (2008: 84) is a survey of contributions to the literature covering the field of probabilistic discounted cash flow (DCF) analysis of individual capital investments from the earliest contributions of the 1960s to today. Sorenson and Lavelle (2008: 42) introduce an approach for comparing the fuzzy set and probabilistic paradigms for ranking vague economic investment information when a present value criterion is used.

## 3. FUZZY SETS AND FUZZY NUMBERS

To deal with vagueness of human thought, Zadeh (1965: 338) first introduced the fuzzy set theory, which was based on the rationality of uncertainty due to imprecision or vagueness. A major contribution of fuzzy set theory is its capability of representing vague knowledge. The theory also allows mathematical operators and programming to be applied to the fuzzy domain.

A fuzzy number is a normal and convex fuzzy set with membership function $\mu_A(x)$ where both satisfys normality: $\mu_A(x)=1$, for at least one $x \in R$ and convexity: $\mu_A(x') \geq \mu_A(x_1) \wedge \mu_A(x_2)$, where $\mu_A(x) \in [0,1]$ and $\forall x' \in [x_1, x_2]$. '$\wedge$' stands for the minimization operator.

Quite often in finance, future cash amounts and interest rates are estimated. One usually employs educated guesses, based on expected values or other statistical techniques to obtain future cash flows and interest rates. Statements like *approximately between* \$12,000 and \$16,000 or *approximately between* 10% and 15% must be translated into an exact amount, such as \$14,000 or 12.5%, respectively. Appropriate fuzzy numbers can be used to capture the vagueness of those statements.

A tilde will be placed above a symbol if the symbol represents a fuzzy set. Therefore, $\tilde{P}$, $\tilde{F}$, $\tilde{G}$, $\tilde{A}$, $\tilde{i}$, $\tilde{r}$ are all fuzzy sets. The membership functions for these fuzzy sets will be denoted by $\mu(x|\tilde{P}), \mu(x|\tilde{F}), \mu(x|\tilde{G})$, etc. A fuzzy number is a special fuzzy subset of the real numbers. The extended operations of fuzzy numbers are given in Appendix A. A triangular fuzzy number (TFN) is shown in Fig. 1. The membership function of a TFN $(\tilde{M})$ is defined by

$$\mu(x|\tilde{M}) = (m_1, f_1(y|\tilde{M})/m_2, m_2/f_2(y|\tilde{M}), m_3), \quad \textbf{(1)}$$

where $m_1 \prec m_2 \prec m_3$, $f_1(y|\tilde{M})$ is a continuous monotone increasing function of $y$ for $0 \leq y \leq 1$ with $f_1(0|\tilde{M}) = m_1$ and $f_1(1|\tilde{M}) = m_2$ and $f_2(y|\tilde{M})$ is continuous monotone decreasing function of $y$ for $0 \leq y \leq 1$ with $f_2(1|\tilde{M}) = m_2$ and $f_2(0|\tilde{M}) = m_3$.

$\mu\left(x\middle|\tilde{M}\right)$ is denoted simply as $(m_1 / m_2, m_2 / m_3)$.

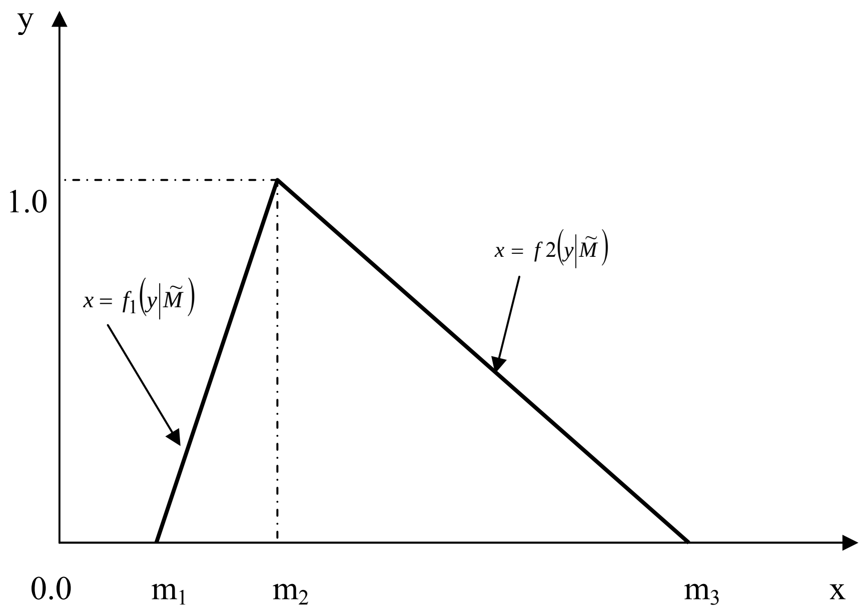


Figure 1. A Triangular Fuzzy Number, $\tilde{M}$

The membership function of a TFN is given by Eq. (2):

$$
\begin{aligned}
\mu\ (x) &= 0, & x < m_1, \\
&= \frac{x - m_1}{m_2 - m_1}, & m_1 \le x \le m_2, \\
&= \frac{m_3 - x}{m_3 - m_2}, & m_2 \le x \le m_3, \\
&= 0, & x > m_3.
\end{aligned} \qquad \textbf{(2)}
$$

A flat (trapezoidal) fuzzy number (FFN) is shown in Fig. 2. The membership function of an FFN, $\tilde{V}$, is defined by

$$\mu\left(x\middle|\tilde{V}\right) = \left(m_1, f_1\left(y\middle|\tilde{V}\right) / m_2, m_3 / f_2\left(y\middle|\tilde{V}\right), m_4\right), \qquad \textbf{(3)}$$

where $m_1 \prec m_2 \prec m_3 \prec m_4$, $f_1\left(y\middle|\widetilde{V}\right)$ is a continuous monotone increasing function of $y$ for $0 \le y \le 1$ with $f_1\left(0\middle|\widetilde{V}\right) = m_1$ and $f_1\left(1\middle|\widetilde{V}\right) = m_2$ and $f_2\left(y\middle|\widetilde{V}\right)$ is continuous monotone decreasing function of $y$ for $0 \le y \le 1$ with $f_2\left(1\middle|\widetilde{V}\right) = m_3$ and $f_2\left(0\middle|\widetilde{V}\right) = m_4$. $\mu\left(x\middle|\widetilde{V}\right)$ is denoted simply as $(m_1/m_2, m_3/m_4)$.

The fuzzy sets $\widetilde{P}$, $\widetilde{F}$, $\widetilde{G}$, $\widetilde{A}$, $\widetilde{i}$, $\widetilde{r}$ are usually fuzzy numbers but n will be discrete positive fuzzy subset of the real numbers (Buckley, 1987: 257). The membership function $\mu(x|\widetilde{n})$ is defined by a collection of positive integers $n_i$, $1 \le i \le K$, where

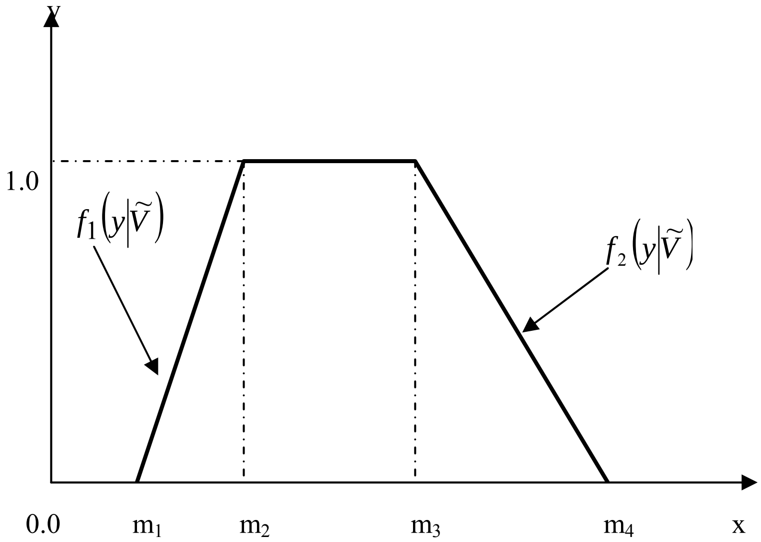


Figure 2. A Trapezoidal (flat) Fuzzy Number, $\widetilde{V}$

$$\mu(x|\widetilde{n}) = \begin{cases} \mu(n_i|\widetilde{n}) = \lambda_i, & 0 \le \lambda_i \le 1, \\ 0 & \textit{otherwise.} \end{cases} \quad \textbf{(4)}$$

The membership function of a FFN is given by Eq. (5)

$$\begin{aligned} \mu(x) &= 0, & x < m_1, \\ &= \frac{x - m_1}{m_2 - m_1}, & m_1 \le x \le m_2, \\ &= 1, & m_2 \le x \le m_3, \\ &= \frac{m_4 - x}{m_4 - m_3}, & m_3 \le x \le m_4, \\ &= 0, & x > m_4. \end{aligned} \quad \textbf{(5)}$$

## 4. FUZZY PRESENT VALUE ANALYSIS

To deal quantitatively with imprecision or uncertainty, fuzzy set theory is primarily concerned with vagueness in human thoughts and perceptions. As an alternative to conventional cash flow models where cash flows are defined as either crisp numbers or risky probability distributions, Chiu and Park (1994: 113) propose an engineering economics decision model in which uncertain cash flows and discount rates are specified as triangular fuzzy numbers. They examine deviation between exact present value (PV) and its approximate form (PVA) and perform the fuzzy project selection by applying different dominance rules as shown in Eqs. (6) and (7), respectively. The result of the exact present value is also a fuzzy number with a non-linear membership function. It is in complex non-linear representations that require tedious computational effort [3]. For the reason of simplicity, a TFN can be used as an approximate form of the complex (exact) present value formula in Eq. (6):.

$$PV = \left[ \sum_{t=0}^{N} \left( \frac{\max\left\{F_t^{l(y)},0\right\}}{\prod_{t'=0}^{t}\left(1+R_{t'}^{r(y)}\right)} + \frac{\min\left\{F_t^{l(y)},0\right\}}{\prod_{t'=0}^{t}\left(1+R_{t'}^{l(y)}\right)} \right), \right.$$

$$\left. \sum_{t=0}^{N} \left( \frac{\max\left\{F_t^{r(y)},0\right\}}{\prod_{t'=0}^{t}\left(1+R_{t'}^{l(y)}\right)} + \frac{\min\left\{F_t^{r(y)},0\right\}}{\prod_{t'=0}^{t}\left(1+R_{t'}^{r(y)}\right)} \right) \right]. \tag{6}$$

where $F_t^{l(y)}$ is the left side representation, $F_t^{r(y)}$ is the right side representation of the fuzzy cash flow $\tilde{F}$ at time t, and $R_{t'}^{l(y)}$ is the left side representation $R_{t'}^{r(y)}$ is the right side representation of the fuzzy interest rate $\tilde{R}$ at time $t'$. N is a crisp number denoting the project life.

When the degree of membership (y) in Eq. (6) is equal to 0, $F_t^{l(y)} = f_{to}$, $F_t^{r(y)} = f_{t2}$, $R_{t'}^{l(y)} = r_{t0}$, $R_{t'}^{r(y)} = r_{t2}$. When the degree of membership (y) in Eq. (6) is equal to 1, $F_t^{l(y)} = F_t^{r(y)} = f_{t1}$, and $R_{t'}^{l(y)} = R_{t'}^{r(y)} = r_{t1}$. Substituting these to the exact present value formula, the approximate form of the present value formula can be derived as in Eq. (7). PVA is represented using its three parameters and it is easier to implement because they are in linear representations.

$$PVA = \begin{bmatrix} \sum_{t=0}^{N} \left( \frac{\max\{f_{t0}, 0\}}{\prod_{t'=0}^{t} (1 + r_{t'2})} + \frac{\min\{f_{t0}, 0\}}{\prod_{t'=0}^{t} (1 + r_{t'0})} \right), \\ \sum_{t=0}^{N} \frac{f_{t1}}{\prod_{t'=0}^{t} (1 + r_{t'1})}, \\ \sum_{t=0}^{N} \left( \frac{\max\{f_{t2}, 0\}}{\prod_{t'=0}^{t} (1 + r_{t'0})} + \frac{\min\{f_{t2}, 0\}}{\prod_{t'=0}^{t} (1 + r_{t'2})} \right) \end{bmatrix} \quad \textbf{(7)}$$

Chiu and Park [3] compute the maximum deviation as a measure of the fitness between PV and PVA. They use very small increments of y as the measurement method instead of derivative method since the latter is difficult to calculate. Using simulation software, they calculate the deviations for different ranges of cash flows and discount rates, and find out that the deviations are not significant unless the confident width of discount rate is greater than an absolute range of ±4%. In the real world applications, when the discount rates are usually estimated within the width of ±4%, PVA can be used in project analysis. The deviations of PV and PVA are depicted in Fig. 3.

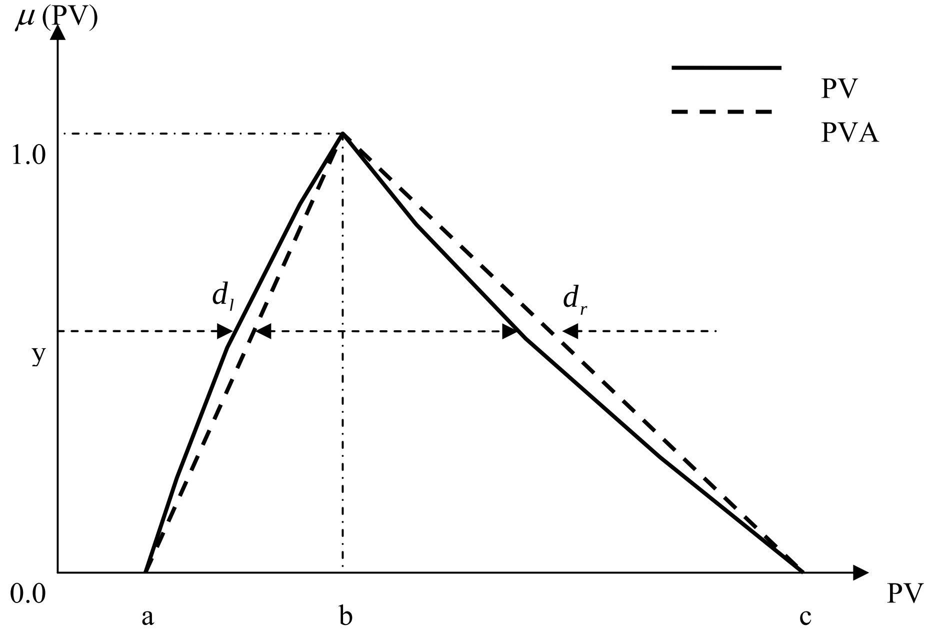


Figure 3. Deviation between PV and PVA

## 5. FUZZY BENEFIT / COST RATIO ANALYSIS

The benefit-cost ratio can be defined as the ratio of the equivalent value of benefits to the equivalent value of costs. The equivalent values can be present values, annual values, or future values. The benefit-cost ratio (BCR) is formulated as

$$BCR = B / C, \tag{8}$$

where *B* represents the equivalent value of the benefits associated with the project and *C* represents the project's net cost (Blank and Tarquin, 1989). A *B*/*C* ratio greater than or equal to 1.0 indicates that the project evaluated is economically advantageous.

In *B*/*C* analyses, costs are not preceded by a minus sign. The objective to be maximized behind the *B*/*C* ratio is to select the alternative with the largest net present value or with the largest net equivalent uniform annual value, because *B*/*C* ratios are obtained from the equations necessary to conduct an analysis on the incremental benefits and costs. Suppose that there are two mutually exclusive alternatives. In this case, for the incremental *BCR* analysis ignoring disbenefits, the following ratios must be used:

$$\frac{\Delta B_{2-1}}{\Delta C_{2-1}} = \frac{\Delta PVB_{2-1}}{\Delta PVC_{2-1}} \tag{9}$$

or

$$\frac{\Delta B_{2-1}}{\Delta C_{2-1}} = \frac{\Delta EUAB_{2-1}}{\Delta EUAC_{2-1}}, \tag{10}$$

where $\Delta B_{2-1}$ is the incremental benefit of Alternative 2 relative to Alternative 1, $\Delta C_{2-1}$ is the incremental cost of Alternative 2 relative to Alternative 1, $\Delta PVB_{2-1}$ is the incremental present value of benefits of Alternative 2 relative to Alternative 1, $\Delta PVC_{2-1}$ is the incremental present value of costs of Alternative 2 relative to Alternative 1, $\Delta EUAB_{2-1}$ is the incremental equivalent uniform annual benefits of Alternative 2 relative to Alternative 1 and $\Delta EUAC_{2-1}$ is the incremental equivalent uniform annual costs of Alternative 2 relative to Alternative 1.

Thus, the concept of *B*/*C* ratio includes the advantages of both NPV and NEUAV analyses.

Because it does not require to use a common multiple of the alternative lives (then

*B*/*C* ratio based on equivalent uniform annual cash flow is used) and it is a more understandable technique relative to rate of return analysis for many financial managers, *B*/*C* analysis can be preferred to the other techniques such as present value analysis, future value analysis, rate of return analysis.

In the case of fuzziness, the steps of the fuzzy *B*/*C* analysis are given in the following (Kahraman et al., 2000: 45):

*Step 1:* Calculate the overall fuzzy measure of benefit-to-cost ratio and eliminate the alternatives that have

$$\tilde{B}/\tilde{C} = \left( \frac{\sum_{t=0}^{n} B_t^{l(y)} \left(1+r^{r(y)}\right)^{-t}}{\sum_{t=0}^{n} C_t^{r(y)} \left(1+r^{r(y)}\right)^{-t}}, \frac{\sum_{t=0}^{n} B_t^{r(y)} \left(1+r^{l(y)}\right)^{-t}}{\sum_{t=0}^{n} C_t^{l(y)} \left(1+r^{l(y)}\right)^{-t}} \right) \prec \tilde{1}, \quad \textbf{(11)}$$

where $\tilde{r}$ is the fuzzy interest rate and r($y$) and l($y$) are the right and left side representations of the fuzzy interest rates and $\tilde{1}$ is (1, 1, 1), and $n$ is the crisp life cycle.

*Step 2:* Assign the alternative that has the lowest initial investment cost as the defender and the next lowest acceptable alternative as the challenger.

*Step 3:* Determine the incremental benefits and the incremental costs between the challenger and the defender.

*Step 4:* Calculate the $\Delta \tilde{B} / \Delta \tilde{C}$ ratio, assuming that the largest possible value for the cash in year $t$ of the alternative with the lowest initial investment cost is less than the least possible value for the cash in year $t$ of the alternative with the next-lowest initial investment cost.

The fuzzy incremental BCR is

$$\frac{\Delta \tilde{B}}{\Delta \tilde{C}} = \left( \frac{\sum_{t=0}^{n} \left(B_{2t}^{l(y)} - B_{1t}^{r(y)}\right)\left(1+r^{r(y)}\right)^{-t}}{\sum_{t=0}^{n} \left(C_{2t}^{r(y)} - C_{1t}^{l(y)}\right)\left(1+r^{r(y)}\right)^{-t}}, \frac{\sum_{t=0}^{n} \left(B_{2t}^{r(y)} - B_{1t}^{l(y)}\right)\left(1+r^{l(y)}\right)^{-t}}{\sum_{t=0}^{n} \left(C_{2t}^{l(y)} - C_{1t}^{r(y)}\right)\left(1+r^{l(y)}\right)^{-t}} \right). \quad \textbf{(12)}$$

If $\Delta\tilde{B}/\Delta\tilde{C}$ is equal or greater than (1, 1, 1), Alternative 2 is preferred.

In the case of a regular annuity, the fuzzy $\tilde{B}/\tilde{C}$ ratio of a single investment alternative is

$$\tilde{B}/\tilde{C} = \left( \frac{A^{l(y)}\gamma\left(n, r^{r(y)}\right)}{C^{r(y)}}, \frac{A^{r(y)}\gamma\left(n, r^{l(y)}\right)}{C^{l(y)}} \right) \quad \textbf{(13)}$$

where $\tilde{C}$ is the first cost and $\tilde{A}$ is the net annual benefit, and $\gamma(n,r) = \left((1+r)^n - 1\right)/\left((1+r)^n r\right)$

The $\Delta\tilde{B}/\Delta\tilde{C}$ ratio in the case of a regular annuity is

$$\Delta\tilde{B}/\Delta\tilde{C} = \left( \frac{\left(A_2^{l(y)} - A_1^{r(y)}\right)\gamma\left(n, r^{r(y)}\right)}{C_2^{r(y)} - C_1^{l(y)}}, \frac{\left(A_2^{r(y)} - A_1^{l(y)}\right)\gamma\left(n, r^{l(y)}\right)}{C_2^{l(y)} - C_1^{r(y)}} \right). \quad \textbf{(14)}$$

*Step 5:* Repeat steps 3 and 4 until only one alternative is left, thus the optimal alternative is obtained.

The cash-flow set $\{A_t = A : t = 1, 2, \ldots, n\}$, consisting of *n* cash flows, each of the same amount as *A*, at times 1,2,…,n, with no cash flow at time zero, is called the equal-payment series. An older name for it is the uniform series, and it has been called an annuity, since one of the meanings of "annuity" is a set of fixed payments for a specified number of years. To find the fuzzy present value of a regular annuity $\{\tilde{A}_t = \tilde{A} : t = n\}$, Eq. (15) is used. The membership function $\mu\left(x \middle| \tilde{P}_n\right)$ for $\tilde{P}_n$ is determined by

$$f_{ni}\left(y \middle| \tilde{P}_n\right) = f_i\left(y \middle| \tilde{A}\right)\gamma\left(n, f_{3-i}\left(y \middle| \tilde{r}\right)\right) \quad \textbf{(15)}$$

For i = 1,2 and $\gamma(n,r) = \left(1 - (1+r)^{-n}\right)/r$. Both $\tilde{A}$ and $\tilde{r}$ are positive fuzzy numbers. $f_1(.)$ and $f_2(.)$ show the left and right representations of the fuzzy numbers, respectively.

In the case of a regular annuity, the fuzzy $\tilde{B}/\tilde{C}$ ratio may be calculated as in the following:

The fuzzy $\tilde{B}/\tilde{C}$ ratio of a single investment alternative is

$$\tilde{B}/\tilde{C} = \left( \frac{A^{l(y)}\gamma\left(n, r^{r(y)}\right)}{FC^{r(y)}}, \frac{A^{r(y)}\gamma\left(n, r^{l(y)}\right)}{FC^{l(y)}} \right), \quad \textbf{(16)}$$

where $F\tilde{C}$ is the first cost and $\tilde{A}$ is the net annual benefit.

The $\Delta\tilde{B}/\Delta\tilde{C}$ ratio in the case of regular annuity is

$$\Delta\tilde{B}/\Delta\tilde{C} = \left( \frac{\left(A_2^{l(y)} - A_1^{r(y)}\right)\gamma\left(n, r^{r(y)}\right)}{FC_2^{r(y)} - FC_1^{l(y)}}, \frac{\left(A_2^{r(y)} - A_1^{l(y)}\right)\gamma\left(n, r^{l(y)}\right)}{FC_2^{l(y)} - FC_1^{r(y)}} \right). \quad \textbf{(17)}$$

Up to this point, we assumed that the alternatives had equal lives. When the alternatives have life cycles different from the analysis period, a common multiple of the alternative lives (CMALs) is calculated for the analysis period. Many times, a CMALs for the analysis period hardly seems realistic (CMALs (7, 13) = 91 years). Instead of an analysis based on present value method, it is appropriate to compare the annual cash flows computed for alternatives based on their own service lives. In the case of unequal lives, the following fuzzy $\tilde{B}/\tilde{C}$ and $\Delta\tilde{B}/\Delta\tilde{C}$ ratios will be used:

$$\tilde{B}/\tilde{C} = \left( \frac{PVB^{l(y)}\beta\left(n, r^{l(y)}\right)}{PVC^{r(y)}\beta\left(n, r^{l(y)}\right)}, \frac{PVB^{r(y)}\beta\left(n, r^{r(y)}\right)}{PVC^{l(y)}\beta\left(n, r^{r(y)}\right)} \right), \quad \textbf{(18)}$$

$$\Delta\tilde{B}/\Delta\tilde{C} = \left( \left( \frac{PVB_2^{l(y)}\beta\left(n, r^{l(y)}\right) - PVB_1^{r(y)}\beta\left(n, r^{r(y)}\right)}{PVC_2^{r(y)}\beta\left(n, r^{l(y)}\right) - PVC_1^{l(y)}\beta\left(n, r^{r(y)}\right)} \right), \left( \frac{PVB_2^{r(y)}\beta\left(n, r^{r(y)}\right) - PVB_1^{l(y)}\beta\left(n, r^{l(y)}\right)}{PVC_2^{l(y)}\beta\left(n, r^{r(y)}\right) - PVC_1^{r(y)}\beta\left(n, r^{l(y)}\right)} \right) \right), \quad \textbf{(19)}$$

where PVB is the present value of benefits, PVC is the present value of costs and

$$\beta(n,r) = ((1+n)^n i / ((1+r)^n - 1). \quad \textbf{(20)}$$

## 6. DEFUZZIFICATION METHODS

The final step is to defuzzify the new fuzzy set to obtain a crisp number (quantitative value) that can be communicated easily. Defuzzification is the conversion of a fuzzy quantity to a precise quantity, just as fuzzification is the conversion of a precise quantity to a fuzzy quantity. The output of a fuzzy process can be the logical union of two or more fuzzy membership functions defined on the universe of discourse of the output variable (Ross, 2005). For example, suppose a fuzzy output is comprised of two parts: the first part, $\tilde{C}_1$, a trapezoidal shape, shown in Fig. 4.a, and the second part, $\tilde{C}_2$, a triangular membership shape, shown in Fig. 4.b. The union of these two membership functions, i.e., $\tilde{C} = \tilde{C}_1 \cup \tilde{C}_2$, involves the max operator, which graphically is the outer envelope of the two shapes shown in Figs. 4.a and b; the resulting shape is shown in Fig. 4.c. Of course, a general fuzzy output process can involve many output parts (more than two), and the membership function representing each part of the output can have shapes other than triangles and trapezoids. Further, as Fig. 4.a shows, the membership functions may not always be normal. In general, we can have

$$\tilde{C}_k = \bigcup_{i=1}^{k} \tilde{C}_i = \tilde{C} \quad \textbf{(20)}$$

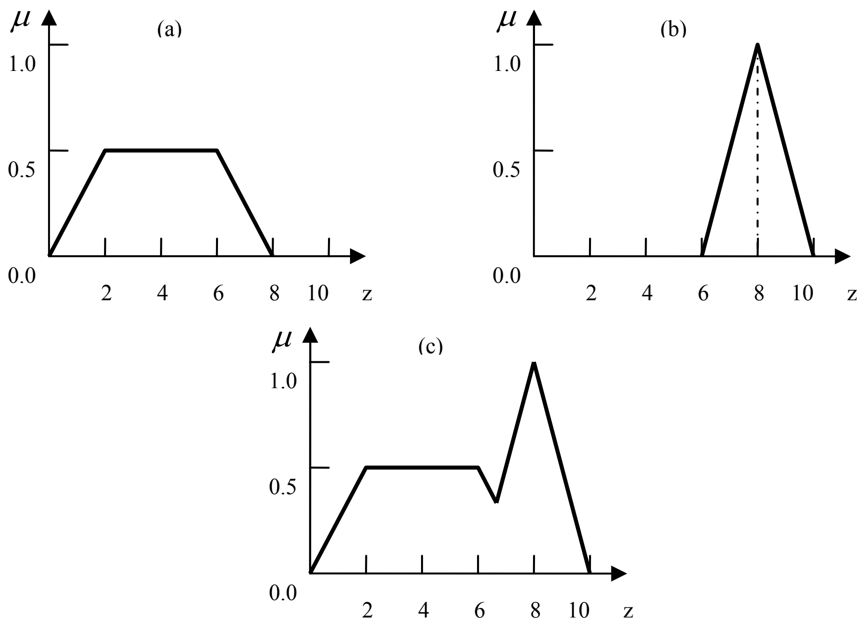


Figure 4. Typical fuzzy process output: (a) first part of fuzzy output; (b) second part of fuzzy output; (c) union of both parts

Among many methods that have been proposed in the literature in recent years, five are described here for defuzzifying fuzzy output functions (membership functions)

*5.1. Max Membership Principle:* Also known as the height method, this scheme is limited to peaked output function. The algebraic expression of this method is given by Eq. (22)

$$\mu_{\tilde{C}}\left(z^*\right) \geq \mu_{\tilde{C}}(z) \qquad \textit{for all } z \in Z \tag{22}$$

where $z^*$ is the defuzzified value, and is shown graphically in Fig. 5.

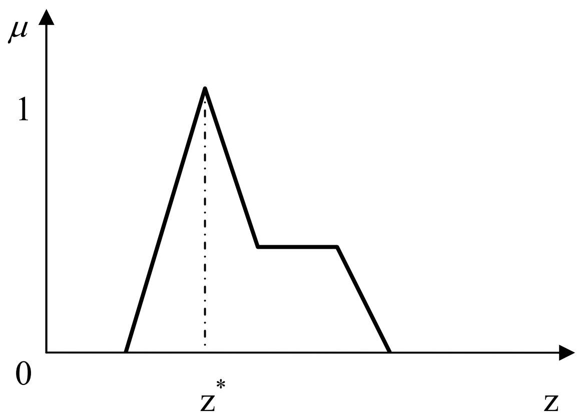


Figure 5. Max membership defuzzification method.

*5.2. Centroid Method:* This procedure (also called center of area, center of gravity) is the most prevalent and physically appealing of all the defuzzification methods (Sugeno; 1985: 59). The algebraic expression of this method is given by Eq. (23)

$$z^* = \frac{\int \mu_{\tilde{C}}(z) \cdot z\, dz}{\int \mu_{\tilde{C}}(z)\, dz} \tag{23}$$

where $\int$ denotes an algebraic integration. This method is shown in Fig. 6.

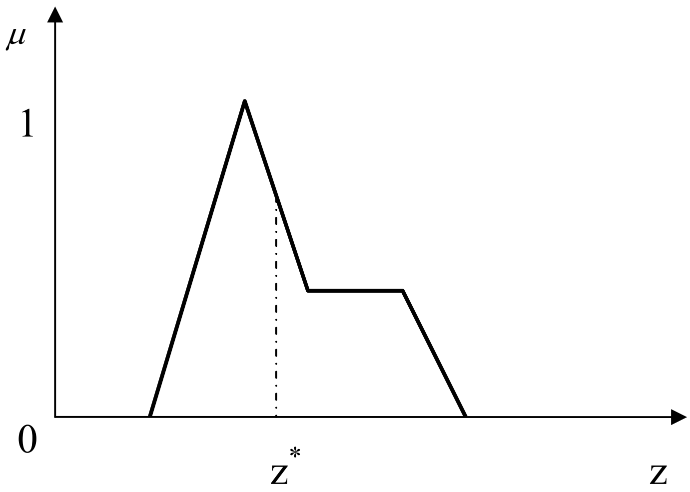


Figure 6. Centroid (COA) method.

*5.3. Weighted average method:* The weighted average method is the most frequently used in fuzzy applications since it is one of the more computationally efficient methods. Unfortunately it is usually restricted to symmetrical output membership functions. The algebraic expression of this method is given by Eq. (24)

$$z^* = \frac{\sum \mu_{\tilde{C}}(\bar{z}) \cdot \bar{z}}{\sum \mu_{\tilde{C}}(\bar{z})} \tag{24}$$

where $\sum$ denotes the algebraic sum and where $\bar{z}$ is the centroid of each symmetric membership function. This method is shown in Fig. 7. The weighted average method is formed by weighting each membership function in the output by its respective maximum membership value. As an example, the two functions shown in Fig. 7 would result in the following general form for the defuzzified value:

$$z^* = \frac{a(0.5) + b(0.9)}{0.5 + 0.9}$$

since the method is limited to symmetrical membership functions, the values a and b are the means (centroids) of their respective shapes.

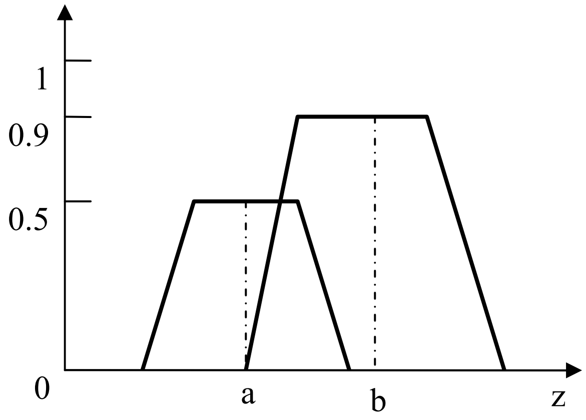


Figure 7. Weighted average method of defuzzification.

*5.4. Mean Max Membership:* This method (also called middle-of-maxima) is closely related to the first method, except that the locations of the maximum membership can be non-unique. The algebraic expression of this method is given by Eq. (25) (Kahraman et al., 2000: 45; Ross, 2005)

$$z^* = \frac{a+b}{2} \quad \textbf{(25)}$$

where *a* and *b* are as defined in Fig. 8.

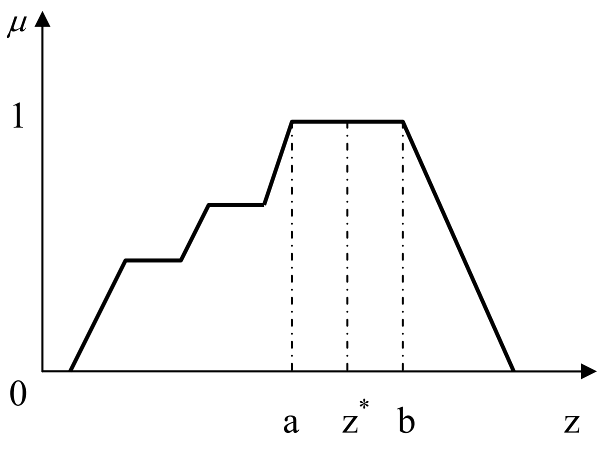


Figure 8. Mean max membership defuzzification method

*5.5 Center of sums:* This is faster than many defuzzification methods that are presently in use, and the method is not restricted to symmetric membership functions.

This process involves the algebraic sum of individual output fuzzy sets, say $\widetilde{C}_1$ and $\widetilde{C}_2$, instead of their union. Two drawbacks to this method are that the intersecting areas are added twice, and the method also involves finding the centroids of the individual membership functions. The defuzzified value $z^*$ is given by Eq. (26)

$$z^* = \frac{\int_z \bar{z} \sum_{k=1}^{n} \mu_{\widetilde{C}_k}(z) dz}{\int_z \sum_{k=1}^{n} \mu_{\widetilde{C}_k}(z) dz} \quad \textbf{(26)}$$

where the symbol $\bar{z}$ is the distance to the centroid of each of the respective membership functions.

This method is similar to the weighted average method, Eq. (24), except in the center of sums method the weights are the areas of the respective membership functions whereas in the weighted average method the weights are individual membership values. Figure 9 is an illustration of the center of sums method.

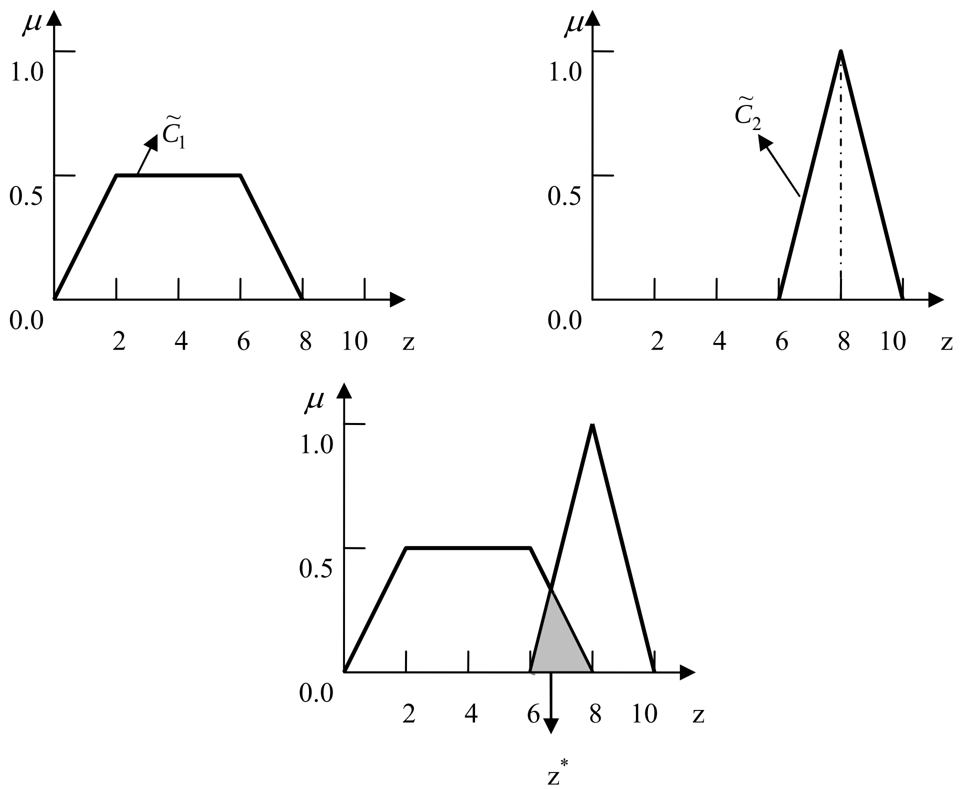


Figure 9. Center of sums method: (a) first membership function; (b) second membership function; (c) defuzzification step.

## 7. A NUMERICAL APPLICATION

In this section, the following problem will be solved by using fuzzy present worth and B/C ratio analyses, respectively.

Let us consider a hypothetical IS project in which it has been estimated that the project life, n = (3,4,5). Project data for year 0 to 5 specified as TFNs are (Omitaomu and Badiru, 2007):

Estimated net cash flow: $F_0$ = (-100,-90,-80); $F_1$ = (-70,-50,-30); $F_2$ = (100,120,140); $F_3$ = (110,140,170); $F_4$ = (130,170,210); $F_5$ = (-100,50,100).

Interest rates: $R_1$=(5%,6%,7%); $R_2$ = (5%,6%,8%); $R_3$ = (5%,7%,10%); $R_4$ = (5%,7%,11%); $R_5$ = (5%,8%,12%).

### *7.1. Fuzzy Present Value Application*

The approximate present value for each parameter of the evaluation period is calculated using Eq. (7). The approximate fuzzy net present value is calculated as follows (Omitaomu and Badiru, 2007):

Using Eq. (7) for n=3:

$$a_1 = 7.6497$$
$$b_1 = 86.0779$$
$$c_1 = 165.2651$$

For n=4:

$$a_2 = 99.7837$$
$$b_2 = 218.2286$$
$$c_2 = 338.0326$$

For n=5:

$$a_3 = 32.2862$$
$$b_3 = 254.2174$$
$$c_3 = 416.3852$$

Hence,

$$PV_1 = (7.6497, 86.0779, 165.2651)$$
$$PV_2 = (99.7837, 218.2286, 338.0326)$$
$$PV_3 = (32.2862, 254.2174, 416.3852)$$

Now the fuzzy PVAs will be aggregated into a single fuzzy set using the *max* method and defuzzify using centroid (COA) method. The *max* method minimizes loss of information; therefore, the idea is to get the maximum combined variability possible in the fuzzy present values. A plot of the three PVAs is shown in Figure 10.

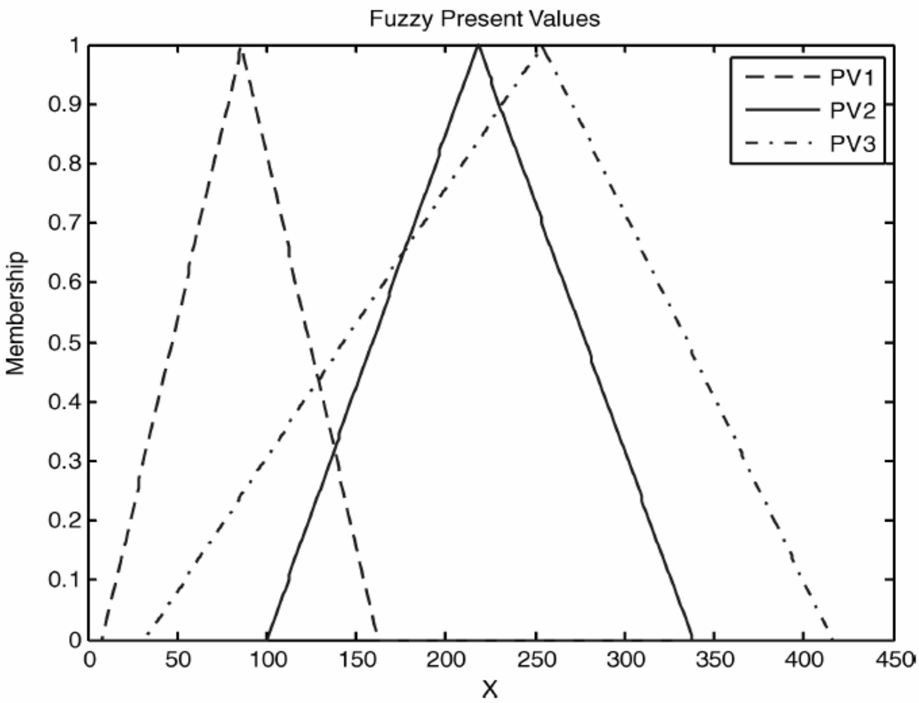


Figure 10. A plot of the three fuzzy present values.

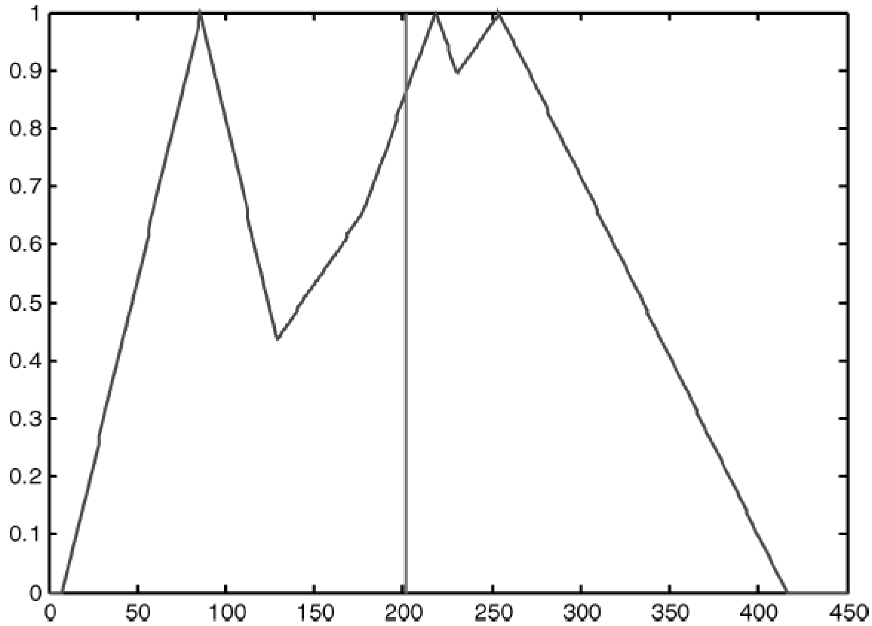


Figure 11. The crisp output using COA

$$z^* = 201.343$$

If we aggregate these plots using the *max* method and defuzzify using centroid (COA) method, we will obtain the plot in Figure 11. As we can see from Figure 11, the combined profile is not a fuzzy number; it is normal but not convex. According to this plot, the precise (crisp) present value for this example is approximately $201.343.

*7.2. B/C Ratio Analysis*

For n= 3,

$$\widetilde{B/C} = (1.0384, 1.6275, 2.5346)$$

For n= 4,

$$\widetilde{B/C} = (1.5963, 2.5909, 4.1338)$$

For n= 5,

$$\tilde{B/C} = (1.1913, 2.8533, 4.8590)$$

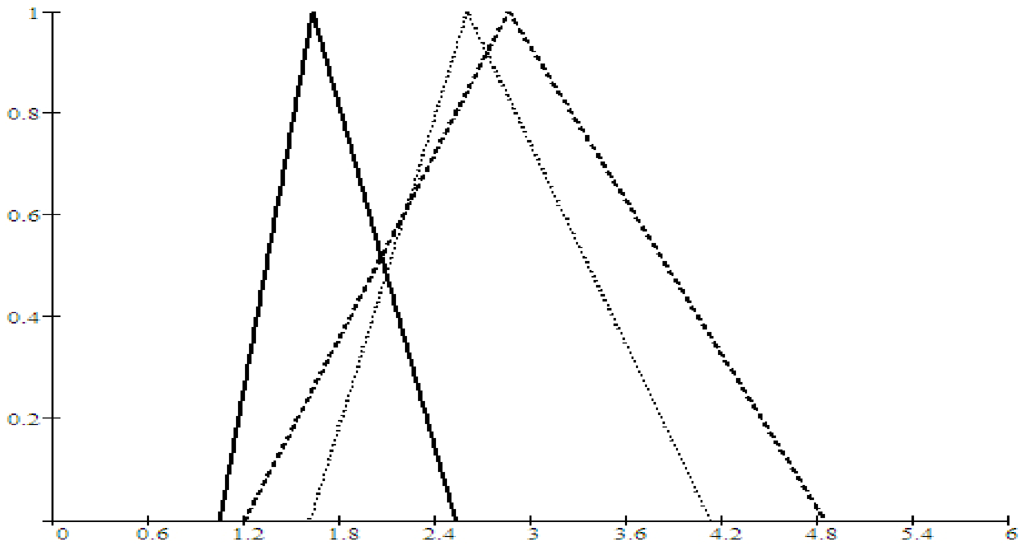

Figure 12. A plot of three B/C ratios.

defuzzifying with centroid (COA) method.

$$z^* = 2.7136$$

## 8. CONCLUSION

Many poor decisions in information system project selection can be attributed to the use of inappropriate evaluation techniques. Information system projects have several unique features that are outstanding from other privately and publicly financed projects. In this paper, a fuzzy present value and B/C ratio analyses have been presented by using triangular fuzzy numbers to take into account the vagueness associated with information system project estimates. Information system projects usually have indefinite cost and benefit streams, indefinite number of evaluation periods, and vague discount rates. Hence, these estimates are highly subjective. The use of triangular fuzzy numbers to model the subjectivity gives a wider range to contain such vagueness. The final fuzzy present value is aggregated and defuzzified using COA to obtain a crisp present value that can be used for comparison purposes. The COA method gives the same ranking with most of the other methods in the literature.

For further research, the other fuzzy capital budgeting techniques like fuzzy rate of return analysis, fuzzy annual cash flow analysis, fuzzy future worth analysis, or fuzzy payback period analysis can be used for the evaluation of IS projects. Besides, some other criteria except cost and benefits can also be incorporated into the IS project selection problem. These criteria may be tangible or intangible. In this case, a fuzzy multicriteria decision making method like TOPSIS, AHP, or ELECTRE can be used for the solution of the problem.

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## Appendix A

One of the most basic concepts of fuzzy set theory which can be used to generalize crisp mathematical concepts to fuzzy sets is the extension principle. Let X be a Cartesian product of universes $X = X_1,\ldots,X_r$, and $\widetilde{A}_1,\ldots,\widetilde{A}_r$ be r fuzzy sets in $X_1,\ldots,X_r$, respectively. $f$ is a mapping from $X$ to universe $Y$, $y = f(x_1,\ldots,x_r)$. Then the extension principle allows us to define a fuzzy set $\widetilde{B}$ in $Y$ by.

$$\widetilde{B} = \{(y, \mu_{\widetilde{B}}(y)) \mid y = f(x_1,\ldots,x_r), (x_1,\ldots,x_r) \in X\}, \qquad \textbf{(A.1)}$$

where

$$\mu_{\widetilde{B}}(y) = \begin{cases} \sup\limits_{(x_1,\ldots,x_r)\in f^{-1}(y)} \min\{\mu_{\widetilde{A}_1}(x_1),\ldots,\mu_{\widetilde{A}_r}(x_r) \in X\}, & \text{if } f^{-1}(y) \neq \varnothing, \\ 0 & \text{otherwise}, \end{cases} \qquad \textbf{(A.2)}$$

where $f^{-1}$ is the inverse of $f$.

Assume $\widetilde{P} = (a,b,c)$ and $\widetilde{Q} = (d,e,f)$, $a,b,c,d,e,f$ are all positive numbers. With this notation and by the extension principle, some of the extended algebraic operations of triangular fuzzy numbers are expressed in the following.

*Changing sign:*

$$-(a,b,c) = (-c,-b,-a)) \quad \textbf{(A.3)}$$

or

$$-(d,e,f) = (-f,-e,-d).) \quad \textbf{(A.4)}$$

*Addition:*

$$\widetilde{P} \oplus \widetilde{Q} = (a+d, b+e, c+f) \quad \textbf{(A.5)}$$

and

$$k \oplus (a,b,c) = (k+a, k+b, k+c) \quad \textbf{(A.6)}$$

or

$$k \oplus (d,e,f) = (k+d, k+e, k+f) \quad \textbf{(A.7)}$$

if $k$ is an ordinary number (a constant).

*Multiplication:*

$$\widetilde{P} \otimes \widetilde{Q} \cong (ad, be, cf) \quad \textbf{(A.8)}$$

and

$$k \otimes (a, b, c) = (ka, kb, kc) \quad \textbf{(A.9)}$$

or

$$k \otimes (d, e, f) = (kd, ke, kf) \quad \textbf{(A.10)}$$

if $k$ is an ordinary number.

*Division:*

$$\widetilde{P} \varnothing \widetilde{Q} \cong (a/f, b/e, c/d). \quad \textbf{(A.11)}$$

The arithmetic operations for two trapezoidal (flat) fuzzy numbers are given in the following. Let $\widetilde{D} = (a, b, c, d)$ and $\widetilde{H} = (e, f, g, h)$ be two positive fuzzy numbers.

*Addition:*

$$\widetilde{D} \oplus \widetilde{H} = (a+e, b+f, c+g, d+h) \quad \textbf{(A.12)}$$

*Subtraction:*

$$\widetilde{D} - \widetilde{H} = (a-h, b-g, c-f, d-e) \quad \textbf{(A.13)}$$

*Multiplication:*

$$\widetilde{D} \otimes \widetilde{H} \cong (a \times e, b \times f, c \times g, d \times h) \quad \textbf{(A.14)}$$

*Division:*

$$\widetilde{D} \varnothing \widetilde{H} \cong (a/h, b/g, c/f, d/e) \quad \textbf{(A.15)}$$